

## How to sketch the graph of $f'$ given the graph of $f$

When first learning to sketch the graph of a derivative function  $f'$ , if you try to do it only by intuition, oftentimes, you will find yourself sketching a graph parallel to  $f$  instead.

### CONCEPTS

1. Where the graph of  $f$  is going upwards to the right, the slope / derivative is positive, so the graph of  $f'$  should be above the  $x$  – axis.

Where the graph of  $f$  is going downwards to the right, the slope / derivative is negative, so the graph of  $f'$  should be below the  $x$  – axis.

2. Where the graph of  $f$  is steeper (\*), the slope / derivative has a large size / absolute value, so the graph of  $f'$  should be farther from the  $x$  – axis.

Where the graph of  $f$  is flatter (\*\*), the slope / derivative has a large size / absolute value, so the graph of  $f'$  should be closer to the  $x$  – axis.

3. Where the graph of  $f$  is a line, the slope / derivative doesn't change, so the graph of  $f'$  should be constant / horizontal.

In particular, where the graph of  $f$  is a horizontal line, the slope / derivative is 0, so the graph of  $f'$  should be 0 / along the  $x$  – axis.

4. Where the graph of  $f$  has a horizontal tangent line, the slope / derivative is 0, so the graph of  $f'$  should be at the  $x$  – axis.

Where the graph of  $f$  has a vertical tangent line or vertical asymptote, the slope / derivative is approaching either positive or negative infinity, so the graph of  $f'$  should have a vertical asymptote.

The behavior of the graph of  $f$  just to the left and right of the vertical tangent line determines if the graph of  $f'$  should be going to positive or negative infinity on each side of the asymptote.

Where the graph of  $f$  has a jump or removable discontinuity, or a cusp, the slope / derivative is undefined, so the graph of  $f'$  should not have a point for that  $x$  – coordinate.

The behavior of the graph of  $f$  just to the left and right of the discontinuity determines if the graph of  $f'$  should be going to a positive or negative number, or to positive or negative infinity on each side of the discontinuity.

(\*) “steep” means rising / falling quickly, like the side of a tall mountain

(\*\*) “flat” means rising / falling slowly, like the ramp in front of the front door of a building

## PROCESS

- A. Identify all  $x$  – coordinates on the graph of  $f$  where there are
- horizontal tangent lines  
the graph of  $f'$  should be at the  $x$  – axis at those  $x$  – coordinates
  - vertical tangent lines or asymptotes  
the graph of  $f'$  should have a vertical asymptote at those  $x$  – coordinates
  - jump or removable discontinuities or cusps  
the graph of  $f'$  should not have a point at those  $x$  – coordinates
- B. Between the  $x$  – coordinates you identified in A
- If the graph of  $f$  is going steeply upwards to the right,  
the graph of  $f'$  should be farther above the  $x$  – axis
  - If the graph of  $f$  is going slowly upwards to the right,  
the graph of  $f'$  should be closer above the  $x$  – axis
  - If the graph of  $f$  is going steeply downwards to the right,  
the graph of  $f'$  should be farther below the  $x$  – axis
  - If the graph of  $f$  is going slowly downwards to the right,  
the graph of  $f'$  should be closer below the  $x$  – axis

As the graph of  $f$  changes its behavior,  
the graph of  $f'$  should

be above the  $x$  – axis as the graph of  $f$  goes upwards to the right  
be below the  $x$  – axis as the graph of  $f$  goes downwards to the right

AND

move farther from the  $x$  – axis as the graph of  $f$  gets steeper  
move closer to the  $x$  – axis as the graph of  $f$  gets flatter